Some Points in Spaces of Small Weight

István Juhász and Kenneth Kunen

October 25, 2001

Abstract
There is a compact 0-dimensional Hausdorff space $X$ of weight $\aleph_1$ with an $x \in X$ which is a weak $P$-point and not a $P$-point. There is a zero-dimensional $L_{\aleph_1}$ space $X$ of weight and cardinality $\aleph_2$, with a non-isolated weak $P_{\aleph_2}$-point to which no discrete subset of $X$ accumulates.

1 Introduction
In this paper, we obtain two examples of spaces of weight $\kappa^+$ where the known example from the literature has weight $2^\kappa$. Both examples involve weak $P_{\kappa^+}$-points that are not $P_{\kappa^+}$-points:

Definition 1.1 For a point $x$ in the topological space $X$:
1. $x \in X$ is a $P_\kappa$-point in $X$ iff the intersection of any family of fewer than $\kappa$ neighbourhoods of $x$ is also a neighbourhood of $x$.
2. $x \in X$ is a weak $P_\kappa$-point in $X$ iff $x$ is not a limit point of any subset of $X \setminus \{x\}$ of size less than $\kappa$.
3. “$P$-point” and “weak $P$-point” mean “$P_{\aleph_1}$-point” and “weak $P_{\aleph_1}$-point”, respectively.

So, in any $T_1$ space, every $P_\kappa$-point is a weak $P_\kappa$-point. If $w(X) = \aleph_0$, then every weak $P$-point is isolated, whereas the ordinal $\omega_1 + 1$ is an example of a space of weight $\aleph_1$ with a non-isolated $P$-point. In Section 2, we shall show:

*2000 Mathematics Subject Classification: Primary 54D20, 54D30.
†Author supported by OTKA Grant no. 25745.
‡Research done while a guest of the Rényi Institute and the Central European University. Support also received from NSF Grant DMS-0097881.
1 INTRODUCTION

Theorem 1.2 There is a compact 0-dimensional Hausdorff space $X$ of weight $\aleph_1$ with an $x \in X$ which is a weak $P$-point and not a $P$-point.

By [7], there is an example of weight $2^{\aleph_0}$, taking $X = \mathbb{N}^* = \beta \mathbb{N}\setminus \mathbb{N}$. To prove Theorem 1.2 in ZFC, we shall apply an elementary submodel argument to this $x \in \mathbb{N}^*$; see Dow [3] for more on such arguments. The point in [7] was a weak $P$-point because it was $\omega_1$-OK (see Definition 2.2). After applying the elementary submodel, the $x$ from Theorem 1.2 will be $\omega_1$-soso, a weaker property which still implies “weak $P$-point”. The strengthening of Theorem 1.2 in which $x$ is actually $\omega_1$-OK is independent of ZFC + $\neg CH$ (see Theorems 2.7 and 2.8).

The following is easy to prove (see, e.g., [4]):

Proposition 1.3 If $X$ is compact Hausdorff and $x \in X$ is not isolated, then $x$ is the accumulation point of some discrete subset.

So, the $x$ of Theorem 1.2 must be a limit of a discrete subset of size $\aleph_1$. However, Proposition 1.3 fails in non-compact spaces:

Theorem 1.4 (van Douwen [1]) There is a countable 0-dimensional Hausdorff space $X$ of weight $2^{\aleph_0}$ with a non-isolated point $p$ to which no discrete subset of $X$ accumulates.

In fact, in this example, $X$ was countable and dense in itself, but every discrete subspace of $X$ was closed, so $p$ could be any point of $X$.

Again, one can ask if the $X$ of Theorem 1.4 can have weight $\aleph_1$. It can, assuming an $L$ space:

Definition 1.5 $X$ is an $L_\kappa$ space iff $X$ is $T_3$ and hereditarily $\kappa$-Lindelöf but not hereditarily $\kappa$-separable. An $L$ space is an $L_\omega$ space.

So, an $L$ space is hereditarily Lindelöf but not hereditarily separable. In Section 3, we shall show:

Theorem 1.6 If there is a 0-dimensional $L_\kappa$ space, then there is a 0-dimensional $L_\kappa$ space $X$ of weight and cardinality $\kappa^+$, with a non-isolated point $p$ to which no discrete subset of $X$ accumulates. Furthermore, $p$ is a weak $P_{\kappa^+}$-point.

Some remarks:

The $p$ in Theorem 1.6 cannot be a $P_{\kappa^+}$-point, since in a $T_3$ hereditarily $\kappa$-Lindelöf space, every point is the intersection of at most $\kappa$ of its neighbourhoods.
2 SOME FLAVOURS OF WEAK P-POINTS

For $\kappa = \omega$, the $X$ in Theorem 1.6 cannot be countable, as it is in Theorem 1.4, since under $MA$, every non-isolated point in a countable $T_2$ space of weight less that $2^{\omega_0}$ is a limit of a discrete $\omega$-sequence.

For $\kappa = \omega$, it is still unknown whether there is an $L$ space in $ZFC$, although there is one in every known model of set-theory. Theorem 1.6 for $\kappa = \omega$ was proved in [4] by a different method which does not seem to generalise to arbitrary $\kappa$. As is well known (see e.g. [5] or [8]), the existence of an $L$ space implies that of a 0-dimensional one of weight $\omega_1$. It is not clear whether this generalises to arbitrary $L_\kappa$ spaces, although once one has a 0-dimensional $L_\kappa$ space, one easily gets one of weight $\kappa^+$ (see Section 3).

For $\kappa = \omega_2$: The existence of a 0-dimensional $L_{\omega_1}$ space is provable in $ZFC$, using Shelah’s colouring theorem; see [9] and Theorem 1.11 of [6]. Thus:

Corollary 1.7 There is a 0-dimensional $L_{\omega_1}$ space $X$ of weight and cardinality $\omega_2$, with a non-isolated point $p$ to which no discrete subset of $X$ accumulates. Furthermore, $p$ is a weak $P_{\omega_2}$-point.

2 Some Flavours of Weak P-Points

As stated in the Introduction, we plan to start with an $x \in \mathbb{N}^*$ which is a weak $P$-point and not a $P$-point, and take an elementary submodel. To compare $x$ in the universe, $V$, with $x$ in the submodel, it is simpler to view $\mathbb{N}^*$ as a Stone space. If $\mathcal{A}$ is a boolean algebra, let $\text{st}(\mathcal{A})$ denotes its Stone space; so $x \in \text{st}(\mathcal{A})$ if $x$ is an ultrafilter on $\mathcal{A}$. The clopen sets of $\text{st}(\mathcal{A})$ are all of the form $N_a = \{x \in \text{st}(\mathcal{A}) : a \in x\}$, for $a \in \mathcal{A}$, so $w(\text{st}(\mathcal{A})) = \mathcal{A}$ whenever $\mathcal{A}$ is infinite. $\mathbb{N}^* = \text{st}(\mathcal{P}(\omega)/\text{fin})$, where $\text{fin} \subset \mathcal{P}(\omega)$ denotes the ideal of finite sets.

Suppose that $x \in \text{st}(\mathcal{A})$ and $x, \mathcal{A} \in M \prec H(\theta)$. Then $x \cap M$ is an ultrafilter on the boolean algebra $\mathcal{A} \cap M$; that is $(x \cap M) \in \text{st}(\mathcal{A} \cap M)$. If $|M| = \aleph_1$, then $w(\text{st}(\mathcal{A} \cap M)) \leq \aleph_1$. Now, we need to relate properties of $x \in \text{st}(\mathcal{A})$ to properties of $(x \cap M) \in \text{st}(\mathcal{A} \cap M)$. The property “not a $P$-point” is easy; $M$ must contain an $\omega$-sequence $\langle N_n : n \in \omega \rangle$ which refutes “$P$-point”, so:

Lemma 2.1 If $x \in \text{st}(\mathcal{A})$ is not a $P$-point in $\text{st}(\mathcal{A})$ and $x, \mathcal{A} \in M \prec H(\theta)$, then $x \cap M$ is not a $P$-point in $\text{st}(\mathcal{A} \cap M)$.

However, the property “weak $P$-point” is trickier. Suppose that $x \in \text{st}(\mathcal{A})$ is a weak $P$-point in $\text{st}(\mathcal{A})$ and is not isolated (i.e., is not a principal ultrafilter generated by an atom). If $M$ is countable, then $\text{st}(\mathcal{A} \cap M)$ will be second
countable and hence separable, so that \( x \cap M \) will not be a weak \( P \)-point in \( \text{st}(A \cap M) \). Even when \(|M| = \aleph_1 \), if \( MA + \neg CH \) holds and \( A \) has the countable chain condition (ccc), then \( \text{st}(A \cap M) \) will still be separable, so that \( x \cap M \) will again fail to be a weak \( P \)-point. Furthermore, there are many examples of such \( x, A \), since under \( MA \) there are weak \( P \)-points in \( \text{st}(A) \) whenever \( A \) is complete and ccc and \( \text{st}(A) \) is not separable (see Dow [2], Theorems 2.3 and 3.2).

Thus, if this plan for proving Theorem 1.2 is to work, we must use a property of \( x \) which implies weak \( P \)-point and which is incompatible with the \( \text{ccc} \). So, we turn to OK points:

**Definition 2.2** For a point \( x \) in a space \( X \):

1. A sequence of neighbourhoods of \( x \), \( \langle U_n : n \in \omega \rangle \), is an \( \omega_1 \)-OK sequence iff there are neighbourhoods \( V_\alpha \) of \( x \) for \( \alpha < \omega_1 \) such that for all \( n \geq 1 \) and all \( \alpha_1 < \cdots < \alpha_n < \omega_1 \), we have \( V_{\alpha_1} \cap \cdots \cap V_{\alpha_n} \subseteq U_n \).
2. \( x \) is \( \omega_1 \)-OK in \( X \) iff every \( \omega \)-sequence of neighbourhoods of \( x \) is \( \omega_1 \)-OK.
3. \( x \) is \( \omega_1 \)-soso in \( X \) iff for every countable family \( W \) of neighbourhoods of \( X \), there is an \( \omega_1 \)-OK sequence of neighbourhoods of \( x \), \( \langle U_n : n \in \omega \rangle \), such that \( W \subseteq \{ U_n : n \in \omega \} \).

Clearly, \( \omega_1 \)-OK implies \( \omega_1 \)-soso. The notion of “\( \omega_1 \)-OK” is from [7], and was used there to produce weak \( P \)-points in \( \aleph^*_1 \). Unfortunately (see Theorem 2.8), it is consistent with \( ZFC \) that for all compact \( X \) of weight \( \aleph_1 \), every \( \omega_1 \)-OK point in \( X \) is already a \( P \)-point. Thus, we turn to the more complicated notion of “\( \omega_1 \)-soso” to prove Theorem 1.2. We remark that no ccc \( T_3 \) space can have a non-isolated \( \omega_1 \)-soso point; the proof is the same as the one in [7] for OK points.

**Lemma 2.3** If \( x \) is \( \omega_1 \)-soso in \( X \) and \( H \) is a \( G_\delta \) set containing \( x \), then there are neighbourhoods \( V_\alpha \) of \( x \) for \( \alpha < \omega_1 \) such that \( \bigcap_{n \in \omega} V_{\alpha_n} \subseteq H \) whenever the \( \alpha_n < \omega_1 \) are distinct.

**Proof.** Apply the definition, 2.2.3, to any \( W \) such that \( \bigcap W = H \).

**Lemma 2.4** If \( X \) is a \( T_1 \) space and \( x \in X \) is \( \omega_1 \)-soso, then \( x \) is a weak \( P \)-point.

**Proof.** If \( Y \) is a countable subset of \( X \setminus \{ x \} \), let \( H = X \setminus Y \). If the \( V_\alpha \) are neighbourhoods of \( x \) as in Lemma 2.3, then all but countably many \( V_\alpha \) are disjoint from \( Y \), so \( x \notin \overline{Y} \).

We call \( M \prec H(\theta) \) \( \omega \)-covering iff for all countable \( E \subseteq M \), there is a countable \( F \in M \) such that \( E \subseteq F \). Such an \( M \) of size \( \aleph_1 \) is easily produced as a union of an elementary chain (see [3], §3).
Lemma 2.5 Assume that \( x \in \text{st}(\mathcal{A}) \) is \( \omega_1 \)-soso in \( \text{st}(\mathcal{A}) \) and that \( x, \mathcal{A} \in M \prec H(\theta) \), where \( M \) is \( \omega \)-covering. Then \( x \cap M \) is \( \omega_1 \)-soso in \( \text{st}(\mathcal{A} \cap M) \).

Proof. Let \( \mathcal{W} = \{ W_i : i \in \omega \} \) be a family of neighbourhoods of \( x \cap M \) in \( \text{st}(\mathcal{A} \cap M) \). Choose \( e_i \in x \cap M \) such that \( N_{e_i} \subseteq W_i \). Then, fix a countable \( F \in M \) such that \( \{ e_i : i \in \omega \} \subseteq F \). Since \( x \in M \), we may assume (intersecting with \( x \)) that \( F \subseteq x \cap M \). Now, apply the definition of “soso” to \( \mathcal{W}' = \{ N_a : a \in F \} \).

Proof of Theorem 1.2. Apply Lemmas 2.1 and 2.5 with \( \mathcal{A} = \mathcal{P}(\omega)/\text{fin} \) and \( x \) any \( \omega_1 \)-OK point in \( \text{st}(\mathcal{A}) \) which is not a \( P \)-point (see [7]). Then in \( \text{st}(\mathcal{A} \cap M) \), the point \( x \cap M \) is not a \( P \)-point, but is \( \omega_1 \)-soso, and hence a weak \( P \)-point.

Now, whether Theorem 1.2 can hold with \( x \) an \( \omega_1 \)-OK point depends on the model of set theory. As usual, \( \varnothing \) denotes the least size of a dominating family in \( \omega^\omega \), and \( b \) denotes the least size of an unbounded family; so \( \aleph_1 \leq b \leq \varnothing \leq 2^b \).

We can modify the proof of Lemma 2.5 to get:

Lemma 2.6 Assume that \( x \in \text{st}(\mathcal{A}) \) is \( \omega_1 \)-OK and \( x, \mathcal{A} \in M \prec H(\theta) \), where \( M \) is \( \omega \)-covering and \( (\omega^\omega) \cap M \) is cofinal in \( \omega^\omega \). Then \( x \cap M \) is \( \omega_1 \)-OK in \( \text{st}(\mathcal{A} \cap M) \).

Proof. Now, we start with a sequence, \( \langle N_{a_n} : n \in \omega \rangle \), of neighbourhoods of \( x \cap M \); so each \( a_n \in x \cap M \). We need to get \( b_\alpha \in x \cap M \) for \( \alpha < \omega_1 \) such that \( b_{\alpha_1} \land \cdots \land b_{\alpha_n} \leq a_n \) whenever \( n \geq 1 \) and \( \alpha_1 < \cdots < \alpha_n < \omega_1 \).

Since \( M \) is \( \omega \)-covering, we can get a sequence \( \langle c_n : n \in \omega \rangle \in M \) such that each \( c_n \in x \cap M \) and each \( a_n = c_\varphi(n) \) for some \( \varphi : \omega \to \omega \). Fix \( \psi \in \omega^\omega \cap M \) such that \( \varphi(n) \leq \psi(n) \) for all \( n \). Note that \( \omega_1 \subseteq M \) since \( M \) is \( \omega \)-covering. Since \( \psi \in M \), we can, in \( M \), apply the definition of \( \omega_1 \)-OK to the sequence \( \langle c_0 \land c_1 \land \cdots \land c_\psi(n) : n \in \omega \rangle \) to get \( b_\alpha \in x \cap M \) for \( \alpha < \omega_1 \) such that for all \( n \geq 1 \) and all \( \alpha_1 < \cdots < \alpha_n < \omega_1 \), we have \( b_{\alpha_1} \land \cdots \land b_{\alpha_n} \leq c_0 \land c_1 \land \cdots \land c_\psi(n) \), and hence \( b_{\alpha_1} \land \cdots \land b_{\alpha_n} \leq c_\varphi(n) = a_n \).

In particular, if \( \varnothing = \aleph_1 \), then we can get \( |M| = \aleph_1 \). Hence, analogously to Theorem 1.2, we have:

Theorem 2.7 If \( \varnothing = \aleph_1 \), then there is a compact Hausdorff space \( X \) of weight \( \aleph_1 \) with an \( \omega_1 \)-OK point which is not a \( P \)-point.

We do not know if the converse to this theorem holds, but the hypothesis cannot be weakened to \( \varnothing = \aleph_1 \)!

Theorem 2.8 Assume that \( V[G] \) is an extension of \( V \) by \( \geq \aleph_2 \) Cohen reals. Then in \( V[G] \):
1. \( b = \omega_1 \).

2. In every compact Hausdorff space \( X \) of weight \( \aleph_1 \) every \( \omega_1 \)-OK point is a \( P \)-point.

Proof. (1) holds because the first \( \aleph_1 \) Cohen reals yield an unbounded family of size \( \aleph_1 \). For (2):

First, work in \( V[G] \): Assume that \( z \in X \) is not a \( P \)-point. We must show that it is not \( \omega_1 \)-OK. Following Tychonov, we may assume that \( X \) is a closed subspace of \([-1, 1]^{\omega_1}\), that \( z = \bar{0} \) (the identically 0 sequence), and that "\( P \)-point" is refuted by the neighbourhoods \( U_n = \{ x \in X : |x_0| < 2^{-n} \} \); that is, \( \bar{0} \) is a boundary point of the set \( \{ x \in X : x_0 = 0 \} \) in \( X \). Let \( D \subseteq X \) be dense in \( X \), with \( |D| \leq \aleph_1 \).

Now, since \( |D| \leq \aleph_1 \), it depends on at most \( \aleph_1 \) of the Cohen reals, so by the usual splitting argument, we may (and shall) assume that \( D \in V \).

In \( V \): Let \( \{ B_\alpha : \alpha < \omega_1 \} \) enumerate a local base for \( \bar{0} \) in \([-1, 1]^{\omega_1}\). Assume that each \( B_\alpha \) is a finitely supported product of rational intervals of the form \((-\delta, \delta)\), and that \( B_n = \{ x \in [-1, 1] : |x_n| < 2^{-n} \} \) for \( n < \omega \). Then, in \( V[G] \), and hence also in \( V \), \( \forall \beta \exists \delta < \omega \ [B_\beta \cap D \not\subseteq B_\delta] \).

Again by splitting, \( V[G] = V[f][H] \), where \( f \in \omega^\omega \) is generic over \( V \) using the partial order \( \mathbb{P} = Fn(\omega, \omega) \), and \( H \) adds the rest of the Cohen reals, via some \( Q = Fn(\kappa, \omega) \). We shall show that in \( V[G] \), the neighbourhoods \( U_n = B_{f(n)} \cap X \) establish that \( \bar{0} \) is not \( \omega_1 \)-OK. To do this, it is sufficient to show that there is no \( S \) such that \( \Phi(f, S) \) holds, where \( \Phi(f, S) \) asserts:

\[
S \subseteq \omega_1 \text{ & } |S| = \aleph_1 \text{ & } \\
\forall n \geq 1 \forall \alpha_1 < \cdots < \alpha_n \ \{ \alpha_1, \ldots, \alpha_n \} \subseteq S \rightarrow B_{\alpha_1} \cap \cdots \cap B_{\alpha_n} \cap D \subseteq B_{f(n)}
\]

If, in \( V[G] \), there is an \( S \) satisfying \( \Phi(f, S) \), then, working in \( V[f] \), there is a \( \mathbb{Q} \)-name \( \dot{S} \) and a \( q \in \mathbb{Q} \) such that \( q \forces \Phi(f, \dot{S}) \). Then, in \( V[f] \), we can find an uncountable \( T \subseteq \omega_1 \) and \( q_\alpha \leq q \) for \( \alpha \in T \) such that each \( q_\alpha \forces [\alpha \in \dot{S}] \).

Furthermore, shrinking \( T \), we may assume that \( \{ q_\alpha : \alpha \in T \} \) is centred, which implies that \( \Phi(f, T) \) holds in \( V[f] \). Furthermore, since \( \mathbb{P} \) is countable, there will be an uncountable subset of \( T \) in \( V \). Thus, shrinking \( T \) again, we may assume that \( T \in V \).

Retreating to \( V \), we have a \( \mathbb{P} \)-name \( \dot{f} \) for the generic function and a \( p \in \mathbb{P} \) such that \( p \forces \Phi(\dot{f}, T) \). Assume that \( \text{dom}(p) \subseteq n \), fix \( \alpha_1 < \cdots < \alpha_n \in T \), and fix \( \beta \) with \( B_\beta \subseteq B_{\alpha_1} \cap \cdots \cap B_{\alpha_n} \). Then \( p \forces [B_\beta \cap D \subseteq B_{f(n)}] \), but \( p \) does not determine the value of \( \dot{f}(n) \). Fix \( i \) so that \( B_\beta \cap D \not\subseteq B_i \), and let \( p' \leq p \) with \( p' \forces \dot{f}(n) = i \). Then \( p' \forces [B_\beta \cap D \not\subseteq B_i] \), a contradiction. \( \square \)
3 Proof of Theorem 1.6

Let $Z$ be our $L_\kappa$ space. Since $Z$ is not hereditarily $\kappa$-separable, it contains a sequence which is left separated in type $\kappa^+$. But then we may assume without loss of generality that this sequence is all of $Z$, so that $Z = \langle \kappa^+, \tau \rangle$, where $\tau$ is some topology on $\kappa^+$. “Left separated” means that every initial segment $\alpha \in \kappa^+$ is closed in $Z$, so each final segment $\kappa^+ \setminus \alpha$ is open. Since $Z$ is 0-dimensional and hereditarily $\kappa$-Lindelöf, we can write:

$$\kappa^+ \setminus \alpha = \bigcup \{ U^\alpha_\xi : \xi \in \kappa \} ,$$

where $U^\alpha_\xi$ is clopen in $Z$ for every $\xi \in \kappa$. Let $\tau_0$ be the coarser topology with a base consisting of all finite boolean combinations from $\mathcal{U} = \{ U^\alpha_\xi : \xi \in \kappa \text{ and } \alpha \in \kappa^+ \}$. Then $\tau_0$ is Hausdorff (because $\mathcal{U}$ separates points), hereditarily $\kappa$-Lindelöf (because it is coarser than $\tau$), and not hereditarily $\kappa$-separable (because it is still left separated). But then, we may assume that $\tau_0 = \tau$, so that $Z$ has weight only $\kappa^+$. Let $Y = [\kappa]^{<\omega} \times Z$, where $[\kappa]^{<\omega}$ is discrete, so that $Y$ is a topological sum of $\kappa$ copies of $Z$. For $E \subseteq Y$ and $a \in [\kappa]^{<\omega}$, let $E_a = \{ \alpha : (a, \alpha) \in E \}$. Our space $X$ will be $Y \cup \{ p \}$ where $p \notin Y$ and $Y$ is an open subspace of $X$. So, the topology on $X$ is defined once we define the neighbourhoods of $p$ in $X$. To this end, for any $\alpha \in \kappa^+$ define $W^\alpha \subseteq Y$ such that for each $a \in [\kappa]^{<\omega}$

$$(W^\alpha)_a = \bigcup \{ U^\alpha_\xi : \xi \in a \} .$$

Now let $\{ W^\alpha \cup \{ p \} : \alpha \in \kappa^+ \}$ be a neighbourhood subbase of $p$ in $X$. Each $W^\alpha$ is clopen in $Y$ (because each $(W^\alpha)_a$ is clopen in $Z$), so that $X$ is 0-dimensional. Also, it is easy to see that $Y$ and $X$ are both $L_\kappa$ spaces of weight $\kappa^+$.

Next, to show that $p$ is non-isolated in $X$, we fix any $\alpha_1, \ldots, \alpha_n \in \kappa^+$ and show that $|W^{\alpha_1} \cap \ldots \cap W^{\alpha_n}| = \kappa^+$. To do this, fix any $\beta \in \kappa^+ \setminus \max\{ \alpha_1, \ldots, \alpha_n \}$. Then, for every $1 \leq i \leq n$, choose $\xi_i \in \kappa$ with $\beta \in U^{\alpha_i}_\xi$. Let $a = \{ \xi_1, \ldots, \xi_n \}$. Then, by definition, we have $\beta \in (W^\alpha)_a$ for every $i$, so $(a, \beta) \in W^{\alpha_1} \cap \ldots \cap W^{\alpha_n}$.

Finally, $p$ is a weak $P_{\kappa^+}$-point in $X$ because for every set $S \subseteq [\kappa]^{<\omega}$ there is some $\alpha \in \kappa^+$ with $S \subseteq [\kappa]^{<\omega} \setminus \alpha$, so that $S \cap W^\alpha = \emptyset$. Then, since $Y$ is hereditarily $\kappa$-Lindelöf, every discrete $D \subseteq Y$ has size $\leq \kappa$, so that $p \notin \mathcal{D}$. □

References

REFERENCES


Rényi Alfréd Institute of Mathematics, Hungarian Academy of Sciences, POB 127, H-1364 Budapest, Hungary

Email address: juhasz@renyi.hu

Department of Mathematics, University of Wisconsin, Madison, WI 53706, USA

Email address: kunen@math.wisc.edu

URL: http://www.math.wisc.edu/~kunen