A. Model Theory

A1. Let T be the theory of all models $\langle A, E \rangle$ where E is an equivalence relation. Prove that T is ω-stable.

A2. Let T be a theory in a countable language. Suppose that for some infinite cardinal κ, every model of T of power κ is atomic. Prove that every model of T is atomic.

A3. Prove that every infinite saturated model has a proper elementary submodel to which it is isomorphic.

A4. Give an example of a model $\mathfrak{A}$ for a countable language such that $\mathfrak{A}$ has power ω_1 but every proper elementary submodel of $\mathfrak{A}$ is countable.

B. Set Theory.

B1. Let N be a transitive class containing all the ordinals, such that for each α, $N \cap R(\alpha) \in N$. Assume that $\langle N, \in \rangle$ satisfies the comprehension axiom schema. Prove that $\langle N, \in \rangle$ is a model of ZF.

B2. Assume the axiom of choice and that the union of fewer than 2^{ω} sets of reals of Lebesgue measure 0 is of Lebesgue measure 0. Prove that 2^{ω} is regular.

B3. Outline a proof of the consistency of Lusin's hypothesis $(2^{\omega} = 2^{\omega_1})$ with the axioms of ZFC.

B4. Assume that ZF is consistent. Show that there is a finite subtheory T of ZF such that in ZF it cannot be proved that $T \cup$ "there is an uncountable inaccessible cardinal" is consistent.

C. Recursion Theory.

C1. Let T be a recursively axiomatized theory in a countable language such that T is $\aleph_0$-categorical. Prove that T has a recursive model.

C2. Let A be a Π^1_1 subset of ω. Show that either $0'$ is hyperarithmetical in A or A is hyperarithmetical.

C3. Show that there is a sequence f_α, $\alpha < \omega_1$, of functions mapping ω into ω such that whenever $\alpha < \beta < \omega_1$, f_α is recursive in f_β but f_β is not recursive in f_α.

C4. Let $\{\varphi_0, \varphi_1, \varphi_2, \dots\}$ be an r. e. set of sentences of first order logic (i. e. the set of gödel numbers is r. e.). Prove that there is a recursive set of sentences $\{\psi_0, \psi_1, \psi_2, \dots\}$ such that for each n, ψ_n is logically equivalent to φ_n.

$\psi_n = \bigwedge(\varphi_n)$

$K_n = \text{length}(\psi_{n-1})$

$i \in K_n$

$< \text{length}(\psi_n)$

then