QUALIFYING EXAM IN LOGIC

August, 1990

INSTRUCTIONS: Do any four problems. Use a separate packet of paper for each problem, since not all of your answers will be graded by the same person. If you think a problem has been stated incorrectly, mention this to the proctor and indicate your interpretation in your solution. In such cases, do not interpret the problem in such a way that it becomes trivial.

NOTATION: ω is the set of natural numbers. $\mathbb{R}$ is the set of real numbers and $\mathbb{Q}$ is the set of rational numbers. The universe of a model $\mathfrak{A}$ is denoted by A. If $\mathfrak{A}$ is a model and $X \subseteq A$, $\mathfrak{A}_X$ is the expansion of $\mathfrak{A}$ formed by adding a constant for each $x \in X$. If $A, B \subseteq \omega$, $A \equiv_T B$ means that A is Turing equivalent to B. A' is the jump of A, and $A \oplus B = \{2^a 3^b : a \in A \text{ and } b \in B\}$. ZF is Zermelo-Fraenkel set theory, and PA is Peano arithmetic. $\kappa^{<\kappa} = \bigcup\{\kappa^\alpha : \alpha < \kappa\}$. CCC denotes the countable chain condition.

ELEMENTARY PROBLEMS

E1. Show that in the theory $ZF-\infty$ consisting of all axioms of ZF except the axiom of infinity, the consistency of PA is not provable.

E2. Prove that the complete theory of the model $\langle \mathbf{R},\mathbf{Q},\leq\rangle$ is decidable.

MODEL THEORY

M1. Prove that there exists a saturated dense linear order of cardinality κ if and only if $\kappa=\kappa^{<\kappa}$.

M2. Let T be an $\forall\exists$ theory in a countable language which has infinite models. Prove that T has a model $\mathfrak{A}$ of power 2^{ω} such that whenever $\mathfrak{A}\subseteq\mathfrak{B}\models T$, every countable set of existential formulas with constants from A which is satisfiable in $\mathfrak{B}_A$ is satisfiable in $\mathfrak{A}_A$.

RECURSION THEORY

R1. Prove or disprove: If A and B are r.e., then $A'\oplus B'\equiv_T(A\oplus B)'$.

R2. Let A be hypersimple and define

$$B=(\langle m,n\rangle : m\leq n \text{ or } m\in A\}.$$

Prove that B is hypersimple but not hyperhypersimple.

SET THEORY

S1. If P and Q are partial orderings in a countable model **M** of ZFC such that P is countably closed and Q is CCC in **M**, then Q is CCC in the generic extension of **M** over P.

S2. Prove that there is a subset $S\subseteq\mathbf{R}$ of power 2^{ω} such that it and its compliment meet every uncountable Borel subset of $\mathbf{R}$.