

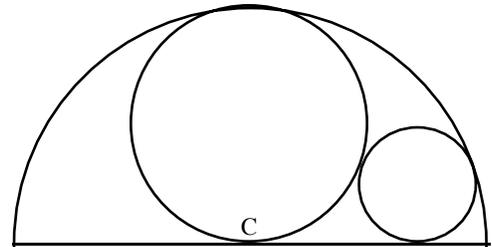
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2001-2002)

OCTOBER 2001

1. For how many positive integers x does there exist a positive integer y with $xy/(x + y) = 100$?

2. In the figure, two circles that are tangent to each other are inscribed in a semicircle of radius 2. If the larger circle is tangent to the diameter of the semicircle at its midpoint C , find the radius of the smaller circle.



3. Let \square be a binary operation defined on the set of nonnegative integers. (This means that if x and y are any two nonnegative integers, then $x \square y$ is a nonnegative integer determined by x and y .) Now suppose that the formula $(x \square y)(y \square z) = x \square z$ holds for all nonnegative integers x, y and z . If $23 \square 47 \neq 0$, compute $61 \square 89$.

4. I have 16 stones, each of which is either red, white or blue, and I place these stones on a 4×4 grid, with one stone at the center of each of the 16 boxes of the grid. Prove that there is at least one stone such that both its row and its column contain another stone of the same color.

5. For which real numbers a does the equation $|x - 1| - 2|x - 2| + 2|x - 3| - |x - 5| = a$ have a unique solution. Here $|x|$ denotes the absolute value of x , defined by $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
 Dept. of Mathematics, 480 Lincoln Drive
 University of Wisconsin, Madison, WI 53706

DEADLINE
 November 2
 2001

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET I