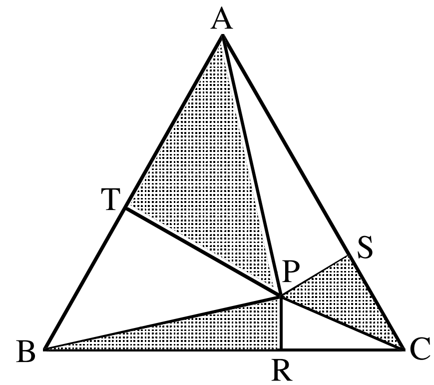
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2001-2002) **NOVEMBER 2001**



1. For how many integers n is the quantity $n^2 - 20n + 244$ equal to a perfect square?

2. In the figure, $\triangle ABC$ is equilateral and P is some point in the interior of the triangle. Perpendiculars $\overline{PR}$, $\overline{PS}$ and $\overline{PT}$ are dropped from P to the sides of the triangle, and lines are drawn from P to the vertices A, B and C. Show that the sum of the areas of the three shaded triangles is exactly half of the area of $\triangle ABC$.

3. Find all positive real numbers x, y and z such that

$$x = \frac{1+z}{1+y} \qquad y = \frac{1+x}{1+z} \qquad z = \frac{1+y}{1+x}.$$

4. Find a positive integer n such that the following is necessarily true: Suppose I have n^2 stones, each of which is either red, white, blue or green, and suppose that I place one of these stones at the center of each of the n^2 boxes of an $n \times n$ square grid. Then there must exist a stone such that both its row and column contain another stone of the same color.

5. Let S be a finite set and recall that two subsets X and Y of S are said to be *disjoint* if they have no elements in common. Suppose that a collection $\mathcal{A}$ of subsets of S has the property that no two of the sets in $\mathcal{A}$ are disjoint but that every subset of S that is not in $\mathcal{A}$ is disjoint from some member of $\mathcal{A}$. Prove that $\mathcal{A}$ contains exactly half of the subsets of S.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE December 3 2001

. . . .

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET II