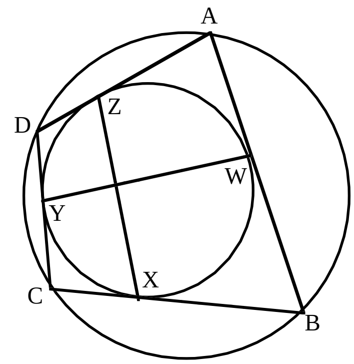
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2001-2002) **FEBRUARY 2002**



1. Find five consecutive positive integers such that the first one is divisible by 5, the next one by 7, the third by 9, the fourth by 11, and the fifth one by 13.

2. Quadrilateral $ABCD$ is inscribed in a circle and also has a circle inscribed in it. If W, X, Y and Z are the points of tangency of the small circle with the sides of the quadrilateral, as shown, prove that $\overline{WY}$ and $\overline{XZ}$ are perpendicular.

3. Let the sequence of integers $a_1, a_2, a_3, \ldots$ be defined by $a_1 = 2$, $a_2 = 3$, $a_3 = 7$, $a_4 = 43$, and in general $a_{n+1} = 1 + a_1 a_2 \cdots a_n$ for all $n \geq 1$. If $s_n = 1/a_1 + 1/a_2 + \cdots + 1/a_n$ and $p_n = 1/(a_1 a_2 \cdots a_n)$ are the sum and the product of the reciprocals of the first n numbers in the sequence, compute $s_{100} + p_{100}$.

4. Does there exist a binary operation $\square$ defined on the set of positive real numbers R^+ such that $(x \square x) \square (x \square x)$ is never equal to $((x \square x) \square x) \square x$ even though $(x \square y) \square x = x \square (y \square x)$ for all $x, y \in R^+$. Recall that $\square$ is a binary operation on R^+ means that, for all x and y in R^+, $x \square y$ is a positive real number determined by x and y.

5. Let $f_1, f_2, f_3, \ldots$ be a sequence of integers satisfying $f_{n-1} + f_n = 2n$ for all $n \geq 2$. If $f_1 = 100$, find f_{1000}.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the page set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE March 15 2002

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V