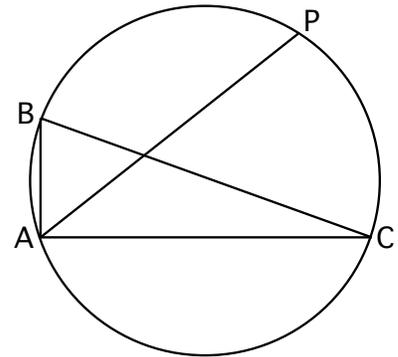


1. Recall that most cubic polynomials have either three distinct real roots or just one such root. Sometimes, however, a cubic has exactly two distinct real roots because two of the three roots coincide. Find all real numbers a such that the polynomial $x^3 - 7x^2 + ax - 9$ has exactly two distinct real roots.



2. In the diagram, $\angle BAC = 90^\circ$. The bisector of this angle is extended to meet the circumcircle of $\triangle ABC$ at point P . Show that $AB + AC = \sqrt{2} AP$.

3. Suppose that 100 on-off light switches are mounted on a control panel in one long line, and that I can flip any batch of consecutive switches simultaneously. Some of the lights are on and some are off, and I want to turn them all off. For example, if lights 1 through 30 are on; lights 31 through 60 are off and lights 61 through 100 are on, I can turn all the lights off in two moves: first flip switches 1 through 30 and then flip switches 61 through 100. Find the smallest number N so that I never need more than N moves to turn off all the lights.

4. Find all solutions in real numbers x , y and z for the equations

$$x^2 + y^2 + z^2 = xy + xz + yz = xyz.$$

5. Does there exist a positive integer n for which it is possible to write $1/n = 1/a^2 + 1/b^2$, where a and b are unequal positive integers? If so, find the smallest such number n .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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