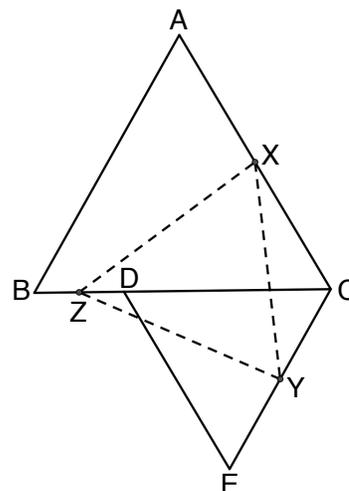


**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
 PROBLEM SET V (2009-2010)**

FEBRUARY 2010

1. Suppose that the integers 1 through 9 are written on a black-board. Erase two of these numbers, which we call a and b , and replace them with the single number $a^2 + 7ab^2 - 3b$. Now there are eight numbers on the board, and we repeat the process: choose two of the eight numbers, call them a and b , and replace them with the one number $a^2 + 7ab^2 - 3b$. Continue like this until just one number remains on the board. Decide whether or not that last remaining number can be 2010.



2. In the diagram, $\triangle ABC$ is equilateral and point D lies on side \overline{BC} . Also, $\triangle CDE$ is equilateral. If points X , Y and Z are the midpoints of \overline{AC} , \overline{CE} and \overline{BD} respectively, show that $\triangle XYZ$ is equilateral.

3. There are 15 committees in an organization, and each committee has nine members. One day, there are a total of 96 people present at committee meetings, and none of these committee meetings is attended by anyone who is not a member. Prove that a majority of the committees have a majority of their members present.

4. Let w , x , y and z be nonnegative, and suppose that $w + x + y + z = 4$. Show that

$$w^2 + x^3 + y^4 + z^5 \geq w + x^2 + y^3 + z^4.$$

5. Find the largest positive integer n such that the set $\{1, 2, 3, \dots, n\}$ can be written as the union of two sets X and Y such that neither X nor Y contains the average of two of its members.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	Deadline March 8, 2010	
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