

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2012-2013)

December 2012

1. In the triangle ABC we have $AB = AC$ and $\angle BAC = 100^\circ$. Let D be the intersection of the bisector at B and the side AC . Show that $BC = BD + DA$.

2. We have 100 points on a circle and connect every point with the other 99. What is the maximum number of intersection points we will have inside the circle?
(For a full solution you have to show that the maximum number can be achieved and that is impossible to get more than that.)

3. Is it possible to find an increasing sequence of 2012 positive integers so that the sum of any two consecutive numbers is equal to the square of the difference of those two numbers?

4. Suppose that a convex polygon has a unit perimeter. Show that it can be covered with a disk of radius $1/4$. (We do not assume that the polygon is regular!)

5. In a ping-pong tournament we had 20 participants. Every player played with everybody else exactly once, and there were no ties. A player X gets a trophy if for every other player Y at least one of the following is true:
 - X beat Y
 - X beat somebody who beat Y .
 Show that there will be at least one trophy awarded.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	Deadline January 1, 2013	
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