

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2012-2013)

January 2013

1. For any two points u and v in the plane, let $d(u, v)$ be the distance from u to v . Let $V = \{v_1, v_2, v_3, v_4, v_5\}$ be a set of five points inside a 2×2 square. Let S be the locus of points p in the plane so that the sum of the squares of the distances from p to each of the five point of V is 50, that is, S is the collection of those points p for which

$$d(p, v_1)^2 + d(p, v_2)^2 + d(p, v_3)^2 + d(p, v_4)^2 + d(p, v_5)^2 = 50.$$

Show that no matter which five points in the square are chosen for the set V , the set S is a circle.

2. At a party we have six married couples. The 12 guests sit down randomly around a large round table. What is the probability that at least one person is not sitting next to his/her spouse? (We assume that when they sit down to the 12 places, each configuration is equally likely.)
3. The set A contains positive integer numbers. We know that if $x, y \in A$ and $x > y$ then $x - y \geq \frac{xy}{16}$. What is the maximal size of A ?
(For full credit you have to show that the maximal size can be achieved, and that you cannot get a larger set satisfying the conditions.)
4. Find all integers a, b, c such that $a^2 + b^2 + c^2 + 2abc = 0$.
5. Let ABC be a triangle with altitudes AD and CE . Show that the perpendicular from B to DE goes through the circumcenter of ABC .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	Deadline February 3, 2013	
Or Email To	talent@math.wisc.edu		
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