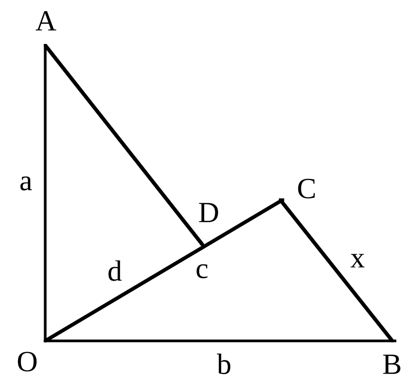
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (1994-95) **NOVEMBER 1994**

1. Suppose $a_0, a_1, a_2, \ldots, a_n$ are positive real numbers satisfying $a_i a_{n-i} = 1$ for all $i = 0, 1, 2, \ldots, n$. If k is any integer, compute the sum

$$\frac{1}{1+a_0^k} + \frac{1}{1+a_1^k} + \frac{1}{1+a_2^k} + \cdots + \frac{1}{1+a_n^k}$$

2. Assume that $\angle AOB$ is a right angle. Find a formula for the area of $\triangle AOD$ in terms of the lengths $OA = a$, $OB = b$, $OC = c$, $OD = d$, and $BC = x$.



3. Let x and y be positive real numbers satisfying the inequality $x^3 + y^3 \le x - y$. Prove that $x^2 + y^2 \le 1$.

4. To each nonnegative integer n, we associate a new integer $f(n)$. Suppose that $f(0) = 0$, $f(1) = 1$, and that for $n \ge 2$ we have $f(n) - 2f(n-1) + f(n-2) = (-1)^n(2n-4)$. Describe $f(n)$ in terms of n.

5. Let us define a process which replaces each 4-tuple of nonzero real numbers $t = (a, b, c, d)$ by a new 4-tuple $t' = (a', b', c', d')$ where $a' = ab$, $b' = bc$, $c' = cd$, and $d' = da$. Suppose we start with a 4-tuple and apply this process again and again. Show that if we ever return to the original 4-tuple, then we must have started with $(1, 1, 1, 1)$.

You are invited to submit a solution even if you get just one problem

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DEADLINE
December 1
1994

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(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

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PROBLEM	SCORE
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