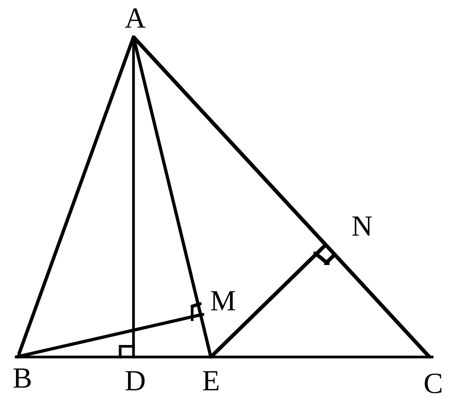
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1994-95) **FEBRUARY 1995**

1. Let $\square$ be an operation (like addition or multiplication) which associates to each pair x, y of real numbers the real number $x \square y$. Suppose that, for all real x, y, z, we have (1) $x \square x = x$, (2) $x \square y = y \square x$, (3) $x \square (y \square z) = (x \square y) \square z$, and (4) if $y < z$ and $x \square y \neq x$, then $x \square y < x \square z$. In the preceding problem set, we showed that $x \square y = x$ or y for all x, y. Find infinitely many different operations $\square$ satisfying the above four conditions.

2. In $\triangle ABC$, suppose that $AD \perp BC$ and that AE is the angle bisector of $\angle BAC$. If $BM \perp AE$ and $EN \perp AC$, prove that points D, M, and N are colinear. (Hint. Use the conclusion of Problem Set IV, Problem 2.)



3. Which positive integers n divide

$$S(n) = 1^{1995} + 2^{1995} + \cdots + (n-1)^{1995}.$$

4. Find all positive integers x and y which satisfy the equation $x^2 + x = y^4 + y^3 + y^2 + y$.

5. An n-digit number α is said to be *special* if (1) α is equal to the arithmetic mean of the $n!$ numbers one obtains by rearranging the digits of α in all possible ways, and (2) the digits of α are not all equal. We know, from the preceding problem set, that the 3-digit special numbers are 370, 407, 481, 518, 592, and 629. Find the next larger special number and then show that there are infinitely many special numbers.

You are invited to submit a solution even if you get just one problem

---

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
March 13
1995

. . . .

(PLEASE DETACH)

LAST NAME FIRST GRADE

SCHOOL TOWN

HOME ADDRESS TOWN ZIP CODE

PROBLEM	**SCORE**
1	
2	
3	
4	
5	