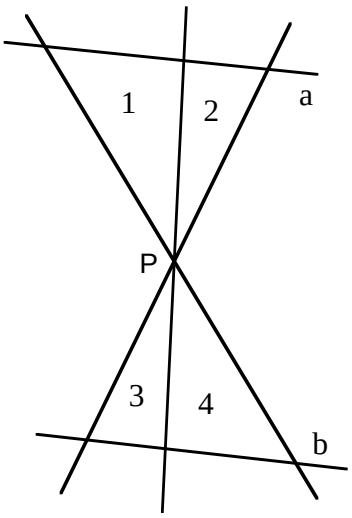
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (1996-97) **OCTOBER 1996**

1. Given real numbers $x,\ a,\ b$ with $x \geq a \geq b \geq 0$, show that

$$\sqrt{x+b} - \sqrt{x-a} \geq \sqrt{x+a} - \sqrt{x-b}$$

2. In the figure, we see three lines through a common point P. These are cut by the two lines a and b as shown, creating four triangular regions, labeled 1, 2, 3 and 4 in the diagram. If the areas of regions 1 and 4 are equal and the areas of regions 2 and 3 are equal, prove that lines a and b are parallel.



3. Let S be a set of positive integers containing 1, 2, 3 and 4. Suppose that for every subset of S consisting of four distinct integers, the sum of that subset is also a member of S. Prove that 1000 is a member of S.

4. Find all positive integers n such that $n^2 + 3n + 1$ is a multiple of $3n + 10$.

5. Find all real numbers x such that

$$\sqrt[3]{x+4} - \sqrt[3]{x} = 1$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE November 1 1996

. . . . . .

(PLEASE DETACH)

LAST NAME	FIRST	GRADE

SCHOOL	TOWN

HOME ADDRESS	TOWN	ZIP CODE

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET I