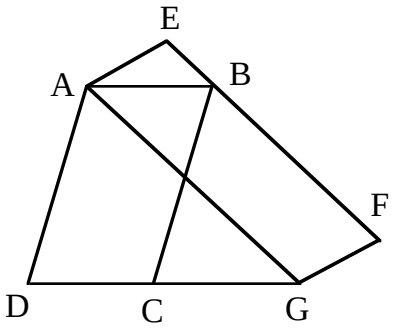
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1996-97) **FEBRUARY 1997**

1. Let $\{a_1, a_2, \ldots, a_{100}\}$ be a set of 100 different real numbers and assume that $a_i > i$ for each subscript i. Now let b_1 be the smallest of the numbers a_i, let b_2 be the second smallest, and so on, so that $\{b_1, b_2, \ldots, b_{100}\}$ is the original set of numbers sorted in such a way that $b_1 < b_2 < \cdots < b_{100}$. Show that $b_i > i$.

2. In the figure, $ABCD$ and $AEFG$ are parallelograms, where B lies on line $\overline{EF}$ and C lies on $\overline{DG}$. Show that the two parallelograms have equal areas.



3. Let $\{x\}$ denote the fractional part of the real number x so that, for example, $\{12/5\} = 2/5$ and $\{3\} = 0$. Find the smallest number x, larger than 1, with $\{x\} + \{1/x\} = 1$.

4. I have a 2 by 9 grid, as indicated, and in each box, I want to write one of the numbers 1, 2, . . . , 9 in such a way that each of these numbers appears twice. I also require that the two occurrences of each number are in boxes that are either horizontally or vertically adjacent. In how many ways can this be done?

5. I compute $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \cdots$, where at the nth step I add the number $\frac{1}{n^3}$. Show that no matter how long I keep this up, my sum will never exceed 1.25.

You are invited to submit a solution even if you get just one problem

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RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
March 14
1997

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(please detach)

Last Name First Grade

School Town

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PROBLEM	SCORE
1	
2	
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PROBLEM SET V