# WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (1997-98)   **DECEMBER 1997**

1. Given two real numbers a and b, suppose that the average of their fourth powers is equal to the fourth power of their average. Show that the two numbers must be equal.

2. Let I be the center of the inscribed circle of $\triangle ABC$. Show that the center of the circumscribed circle of $\triangle BIC$ lies on the circumscribed circle of $\triangle ABC$.

3. Recall that a rational number is one that can be written in the form m/n, where m and n are integers. Suppose that a, b and c are positive rational numbers and that $\sqrt{a} + \sqrt{b} + \sqrt{c}$ is also rational. Show that $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{c}$ are each rational.

4. Suppose that all the integers $n > 1000$ are divided into two sets A and B. Show that at least one of these sets contains two different numbers x and y such that $x + y$ is also in that set.

5. Recall that the Fibonacci numbers are $1, 1, 2, 3, 5, 8, \ldots$, where after the first two, each is the sum of the preceding two numbers. Write F_n to denote the nth Fibonnaci number and let r denote the number $(1 + \sqrt{5})/2$. Prove that the ratio F_{100}/F_{99} is so close to r that the difference satisfies $|F_{100}/F_{99} - r| < 10^{-20}$.
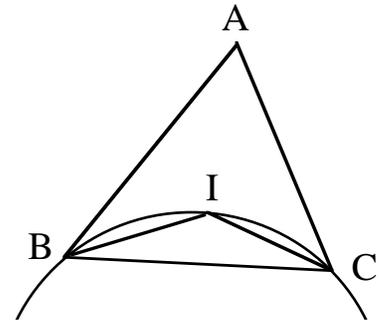
You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH   DEADLINE
                              Dept. of Mathematics, 480 Lincoln Drive   January 9
                              University of Wisconsin, Madison, WI 53706   1998
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(Please Detach)

PROBLEM	SCORE
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2	
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5	

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Last Name   First Name   Grade

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School   Town

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Home Address   Town   Zip Code

PROBLEM SET III