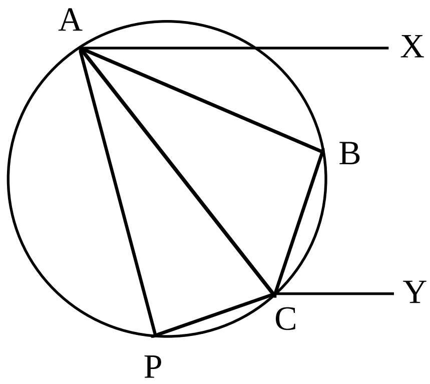
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1997-98) **JANUARY 1998**

1. Suppose that a and b are integers such that $a + 2b$ and $b + 2a$ are squares. Prove that each of a and b is a multiple of 3.

2. In the figure, P is a point on the circumcircle of $\triangle ABC$. Lines $\overline{AX}$ and $\overline{CY}$ are drawn so that $\angle PAC = \angle BAX$ and $\angle PCA = \angle BCY$. Prove that $\overline{AX}$ and $\overline{CY}$ are parallel.



3. (NEW YEAR'S PROBLEM) Let us write $P(n)$ to denote the smallest prime number that does NOT divide n and use $Q(n)$ to denote the product of all prime numbers less than n, with $Q(2)$ defined to be 1. Construct a sequence of numbers X_n as follows. Put $X_0 = 1$ and for each integer $n > 0$, define $X_n = X_{n-1}P(X_{n-1})/Q(P(X_{n-1}))$. Thus the first several numbers in this sequence are 1, 2, 3, 6, 5, 10, 15, 30, 7, ... Compute X_{1998}.

4. Prove that the average of the squares of three real numbers can never be less than the square of the average of these numbers.

5. Find all polynomial functions $F(x)$ such that $F(0) = 2$ and $F(x^2 + 1) = F(x)^2 + 1$ for all x.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE February 13 1998

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
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5	

PROBLEM SET IV