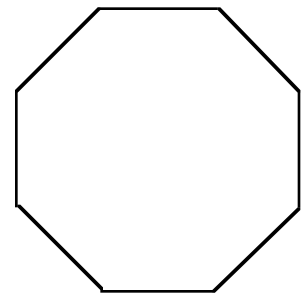
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1997-98) **FEBRUARY 1998**

1. Let r be a real number with $-6 < r < 6$. Find all real numbers a, b, c, d that satisfy $x^4 + rx^2 + 9 = (x^2 + ax + b)(x^2 + cx + d)$, a polynomial equation in the variable x.

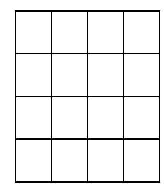
2. Cut a regular octagon into seven pieces so that these pieces can be put together in one way to form a rectangle and then in another way to form a second rectangle of a different shape.



3. Let q be a positive rational number. Show that there are only finitely many positive integers n_1, n_2, n_3, n_4 with

$$\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4} = q.$$

(Of course, there may be no solutions.)



4. We wish to fill the 16 boxes of the 4×4 square array with the letters a, b, c and d in such a way that each letter appears precisely once in each row and precisely once in each column. In how many different ways can this be done?

5. For any positive integer n, let $f(n) = n + 1$ if n is odd and $f(n) = n/2$ if n is even. Now let $g(n)$ be the smallest number of times that the function f must be applied repeatedly to n until 1 is reached. For instance, $g(1) = 0$, $g(2) = 1$, $g(3) = 3$ (since $f(f(f(3))) = 1$), and $g(4) = 2$ (since $f(f(4)) = 1$). Find a general rule for determining $g(n)$ from n.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
March 13
1998

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V