

INTRODUCTORY PROBLEMS (09/20/17)

1. Describe all numbers n such that the number of distinct positive divisors of n is a prime.
2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function such that $f(f(f(f(f(0)))))) = 0$. Show that $f(x) = x$ for some x .
3. (UWUMC'17, #2) Set

$$f(x) = \sum_{k=0}^{\infty} \frac{x+k}{e^k}.$$

Find

$$\int_0^2 f(x)e^{-x} dx.$$

4. (VTRMC'84, #1) Find the units digit (base 10) in the sum

$$\sum_{k=1}^{99} k!$$

5. (VTRMC'89, #4) Let a, b, c, d be distinct integers such that the equation

$$(x-a)(x-b)(x-c)(x-d) - 9 = 0$$

has an integer root r . Show that

$$4r = a + b + c + d.$$

(This is essentially a problem from the 1947 Putnam examination.)

6. (Putnam'02, A1) Let k be a fixed positive integer. The n -th derivative of

$$\frac{1}{x^k - 1}$$

has the form

$$\frac{P_n(x)}{(x^k - 1)^{n+1}},$$

where $P_n(x)$ is a polynomial. Find $P_n(1)$.

UW Putnam Club

Meeting time: Wednesday 5–6:30pm, VV B139. Organizers: Dima Arinkin, Gheorghe Craciun, Mihaela Ifrim.

Putnam competition: First Saturday in December (**December 2**, 2017). Two three-hour sessions of six problems each. Over 2,000 college students participate; there is also an official UW team (3 students).

Virginia Tech Regional Math Competition: 9–11:30 am, **October 21**, 2017, 7 problems. More than 700 contestants from around 100 schools. Kind of ‘Putnam preparation’, somewhat easier.

UW Undergraduate Math Competition: Spring 2018 (probably April).

Typical topics: Linear algebra, elementary number theory, calculus, combinatorics; emphasis on problem-solving.